

### BIG PICTURE

Students will:

- explore and generalize patterns;
- develop an understanding of variables;
- investigate and compare different representations of patterns.

Day	Lesson Title	Description	Expectations
5	Toothpick Patterns	<ul style="list-style-type: none"> <li>• Review patterning concepts.</li> <li>• Build a growing pattern.</li> <li>• Explore multiple representations.</li> </ul>	7m70, 7m72 CGE 3c, 4f
6	Patterns with Tiles	<ul style="list-style-type: none"> <li>• Build a pattern.</li> <li>• Introduce the <math>n</math>th term.</li> </ul>	7m66, 7m71 CGE 4b
7	Pattern Practice	<ul style="list-style-type: none"> <li>• Continue development of patterning skills.</li> </ul>	7m67, 7m71, 7m75 CGE 2c, 5e
8	Pattern Exchange	<ul style="list-style-type: none"> <li>• Share work from previous day.</li> </ul>	7m69, 7m75 CGE 2c, 5e
9	Performance Task	<ul style="list-style-type: none"> <li>• Complete the performance task.</li> </ul>	7m66, 7m67, 7m73, 7m75 CGE 5g

**Description**

- Review patterning concepts.
- Build a growing pattern.
- Explore multiple representations.

**Materials**

- toothpicks
- BLM 5.1

**Assessment Opportunities****Minds On ...****Whole Class → Discussion**

Students contribute to a concept map about patterning, built on the board. Based on their experiences with patterning, they may identify types of patterns, materials for patterns, sample numerical or geometrical patterns, or applications of patterns.

Discuss why the ability to identify and discuss patterns is important. Ask a student to present a pattern on the board and another student to draw a different type of pattern. Have other students add the next term to each pattern and explain thinking. Use this as a sample to complete the understanding column in BLM 5.1. (See *Patterning to Algebraic Modelling* – Section 2)

**Curriculum Expectations/Observation/Mental Note:** Assess students' understanding of patterns, their confidence in using them, and their use of mathematical language.

Distinguish between a growing or diminishing "pattern" and a constant "design."

People use patterns to investigate and represent complex relationships existing in many areas, including nature and science.

**Action!****Pairs → Activity**

On an overhead, create the first two terms of the toothpick pattern presented on BLM 5.1. Ask a student to create the third term of the pattern.

In pairs, students continue the pattern with their toothpicks, and complete BLM 5.1. Circulate, offering assistance as required. Encourage students to look at different ways to build the "5<sup>th</sup> term," the "25<sup>th</sup> term," the "*n*th term," etc.

There is no "right way" to formulate the construction of a term.

Students discuss solutions with their partners. Stress that each partner may have a different entry in the "understanding" column but should have the same value in the "number of toothpicks" column.

See TIP 1 – Multiple Representations – Pattern Building for examples of different ways students might see the pattern.

**Consolidate Debrief****Whole Class → Discussion**

Students share their approaches. Discuss different entries in the "understanding" column, highlighting the validity of all representations. Students should be using words and numbers but may not be using variables at this point.

It is important that students understand the limitation of recursive representations, e.g., add three to the last term. They should be encouraged to move to more sophisticated patterning, e.g., predicting the number of toothpicks required by consideration of the term number.

Each student should complete BLM. 5.1 during this class.

**Home Activity or Further Classroom Consolidation**

Design another toothpick pattern, building and recording the first three terms. Explain your pattern and consider how many toothpicks would be required to build the 100<sup>th</sup> term in the pattern.

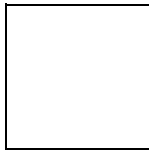
*Concept Practice*

## 5.1: Toothpick Patterns

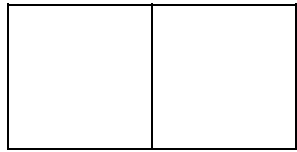
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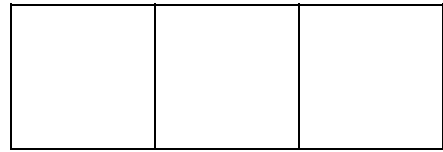
1. Build this pattern with toothpicks.



Term 1



Term 2



Term 3

2. Build the next two terms in the pattern.

3. Complete the chart. Put a numerical explanation of the number of toothpicks required in the Understanding column.

Term	Number of Toothpicks	Understanding
1		
2		
3		
4		
5		

4. How many toothpicks would you require to build the 100<sup>th</sup> term? Explain your thinking.

5. Explain how to build the 100<sup>th</sup> term another way.

**Description**

- Build a pattern.
- Introduce the  $n$ th term.

**Materials**

- colour tiles
- BLM 6.1

**Assessment Opportunities****Minds On ...****Whole Class → Discussion**

Review concepts of patterns, and the idea that pattern building can be expressed in different ways. Selected students share patterns that they developed. Other students predict the number of toothpicks required for the 100<sup>th</sup> term (BLM 5.1) and share their responses and strategies.

Encourage students to think about how they built each term in the pattern.

**Curriculum Expectations/Observation/Mental Note:** Collect diagnostic information during the discussion:

- which students are developing and using simplistic patterns
- which are developing more complex patterns
- which students are ready to move to variables.

Using a variable in reference to a term number contrasts use of a variable as a placeholder for a single unknown value in a linear equation.

Not all students will be ready for the former use of a variable in Grade 7, but all students should be comfortable with the latter use in Grade 7.

**Action!****Pairs → Activity**

Based on observations from the class discussion, pair students homogeneously according to their development level to allow for targeted assistance during the activity.

Student pairs complete BLM 6.1. Circulate, encouraging each partner to share her/his description.

**Consolidate Debrief****Whole Class → Discussion**

Ask a group to model on the board or overhead one representation for the tile pattern, writing their pattern clearly in words. Some students may choose to use variables. Encourage students to compare the various descriptions of the pattern, as well as the different representations (words versus algebraic) and discuss the advantages of each. Students should see that the various descriptions all represent the same situation and should look for connections between the descriptions.

Select a student to add vocabulary to the Word Wall - *term, term number, variable*.

**Home Activity or Further Classroom Consolidation**

Revisit your toothpick patterns. Find two other ways to express your pattern. Consider other rules for generating the same pattern and/or express the pattern using variables, if appropriate.

Discuss the use of patterns with an adult. In your math journal, describe how they use patterns in their career or hobbies.

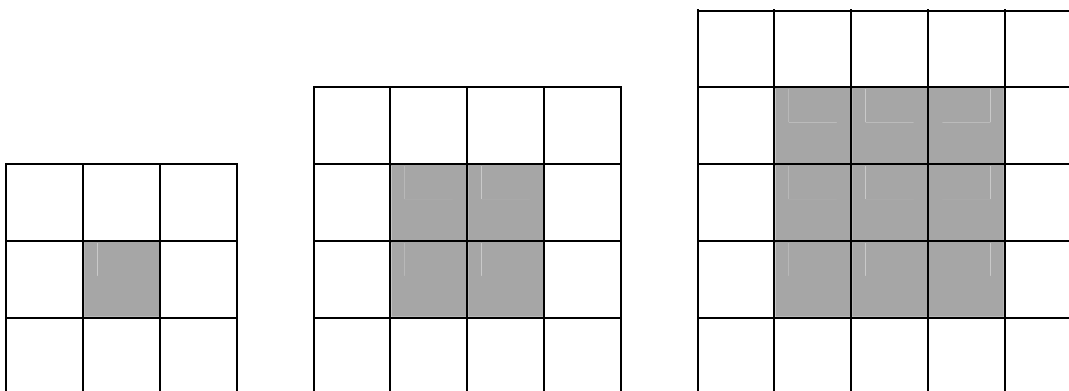
*Concept Practice  
Exploration  
Skill Drill*

## 6.1: Patterns with Tiles

Name:

Date:

1. Build the first five terms of this sequence using light and dark tiles.



2. Complete the table.

Term Number	Number of Light Tiles	Understanding	Number of Dark Tiles	Understanding
1				
2				
3				
4				
5				

3. a) How many dark tiles are there in the 10<sup>th</sup> term? Explain your reasoning.

b) How many light tiles are there in the 10<sup>th</sup> term? Explain your reasoning.

4. How many light tiles are there in the 100<sup>th</sup> term? Explain your reasoning.

5. Describe a strategy for working out how many dark tiles and how many light tiles are needed to build any term.

**Description**

- Continued development of patterning skills.

**Materials**

- manipulatives, e.g., tiles, toothpicks
- BLM 7.1, 7.2
- Developing Mathematical Processes questions

**Assessment Opportunities****Minds On ...****Groups of 4 → Placemat**

To heighten their awareness of linkages between mathematics and life experiences, students share the ideas they determined about using patterns in jobs and hobbies.

**Action!****Small Group → Activity**

**Curriculum Expectations/Exhibition/Checklist:** Circulate and observe students as they complete BLM 7.1. Assess their ability to recognize a pattern, extend a pattern, express the pattern using words and/or algebraically, and determine a value for a non-consecutive term (such as 100<sup>th</sup> term).

Students work in groups of three to complete BLM 7.1. Provide assistance, as required.

On chart paper or on the board, students record their responses to b) and c) for each of the three patterns on BLM 7.1. Allow a portion of the class for students to add their method for describing the 20<sup>th</sup> and the  $n$ th terms to the charts/board. Students should put their initials beside their response. Students create their own patterns (BLM 7.2).



Some students may move to abstract representations, while others may continue to use concrete materials.

Refer to TIP 1 – Multiple Representations – Pattern Building

**Consolidate Debrief****Pairs → Sharing**

Students read over the responses from one section of BLM 7.1, such as 1b. Allow for some individual interpretation. Facilitate a dialogue between the students, encouraging them to ask clarifying questions of each other.

Note: Students may respond using a variety of representations (words and/or algebraic expressions). Students' descriptions of their pattern and their representations may vary; however their representations should be equivalent. Encourage students to identify equivalent expressions that look significantly different and explain how they determined equivalency.

Students could discuss the equivalency of different representations and expressions, e.g., different but equivalent representations may be “double the number” or “add the number to itself” or “multiply the number by two” or “ $2n$ .”

**Home Activity or Further Classroom Consolidation**

Complete worksheet 7.2. Have an adult answer your three questions. Pay close attention to the process that they use to answer these questions. Record their process in your math journal and identify if it is the same or different from your process.

Complete the questions, extend a pattern, complete a table, and write words to explain the pattern.

See *Patterning to Algebraic Expressions* – Section 2 *Developing Mathematical Processes* p. 5.

*Concept Practice*

## 7.1: Pattern Practice

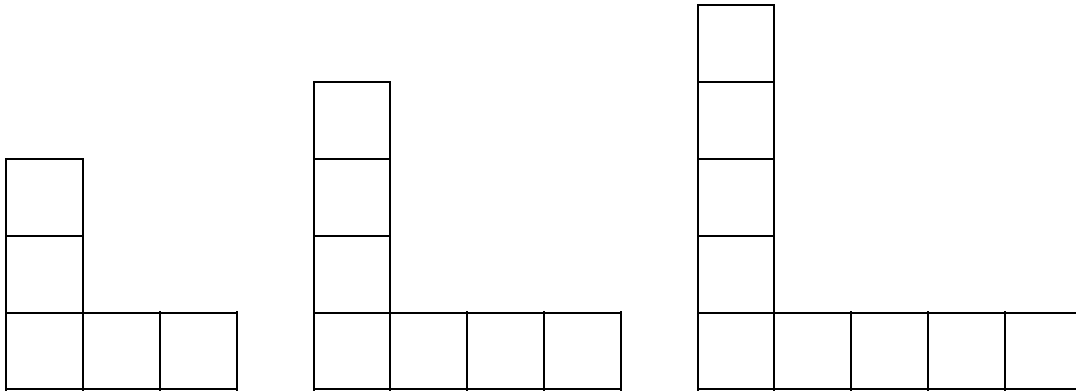
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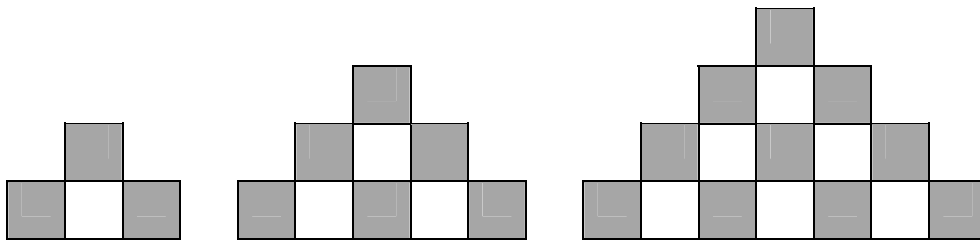
For each example below:

- build the first few terms of the pattern
- write at least two different ways to describe how to build the 20<sup>th</sup> term
- write at least two different ways to describe how to build the  $n$ th term.

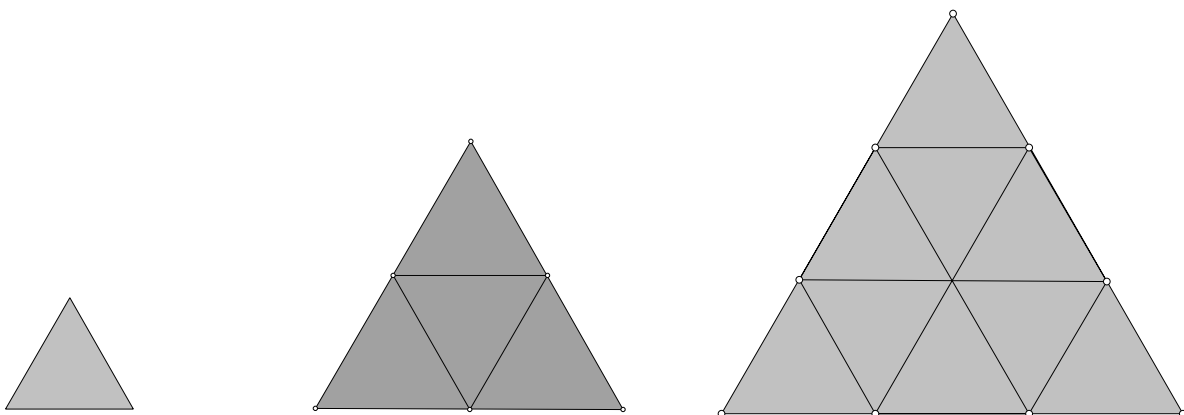
1.



2.



3.



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## 7.2: Pattern Posing

Name:

Date:



Create your own pattern using tiles, toothpicks, or another material.

Materials used: \_\_\_\_\_

A drawing of the first three terms of my pattern

A large empty rectangular box with a black border, intended for drawing a pattern.

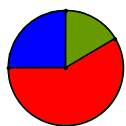
Three questions that someone could answer about my pattern are:

1.

2.

3.



**Description**

- Share work from previous day.

**Materials**

- completed BLM 7.2
- colour tiles, toothpicks

**Assessment Opportunities****Minds On ...****Whole Class → Discussion**

Students share their solutions to the Developing Mathematical Processes questions from Day 1. Validate various correct representations. Select some expressions that are equivalent and have students justify that they are. It is important to value variety in responses so that the stage will be set for algebraic manipulations introduced when one representation is more appropriate for particular applications. Students recognize that some representations should be based on the term number, not the value of the previous term (functional, not recursive).

**Curriculum Expectations/Observation/Anecdotal:** Make a note of students who are still defining patterns recursively in order to provide some direct assistance during the activity.

**Action!****Individual → Activity**

Use a chain to re-distribute students completed BLM 7.2. (See Differentiated Instruction below.) Students record their responses on a paper, with the creator's name and their name at the top. Use a chain to re-distribute questions again so that students respond to the patterns created by two different students.

**Curriculum Expectations/Self-Check/Anecdotal:** Each student should receive their BLM 7.2 and two responses from their peers. They review their peers' work and discuss their solutions as required.

**Differentiated Instruction**

The sharing of BLM 7.2 could happen within three groups in the classroom. The first group could be the students who are comfortable creating general terms and may use algebraic representation. The second group could be the students who encounter slight difficulty with the concept but are usually able to express a general term, perhaps using words. The third group would be the students identified by the teacher in the Minds On section. The teacher may choose to work directly with the third group on the process for generating a rule based on the term number.

**Consolidate Debrief****Whole Class → Discussion**

Students share interesting patterns that they encountered during the lesson and some of the dialogue that occurred during the self-check process.

**Home Activity or Further Classroom Consolidation**

In your math journal, reflect on your individual skill development. Can you:

- extend a pattern?
- describe a pattern in words?
- use a pattern to make a prediction?
- determine a specific term (such as the 100<sup>th</sup> term) by referencing the term number rather than the previous term?
- use appropriate language to describe the pattern?

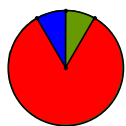
Complete the following questions for additional practice: (The teacher inserts text references.)

*Reflection  
Concept Practice*

By generating multiple responses for one question, the teacher shows that thinking is valued, as well as correct answers.

Students should go to the author for clarification, rather than the teacher.

A general debriefing of the activity allows the teacher to see where the students are in their understanding of patterns.

**Description**

- Complete the performance task.

**Materials**

- BLM 9.1
- colour tiles

**Assessment Opportunities****Minds On ...****Whole Class → Set the Stage**

Explain to students that they will be working individually on an activity that is similar to the ones completed during the past few lessons. Discuss with students the rubric on which they will be assessed.

Students read over BLM 9.1 and ask any clarifying questions.

**Action!****Individual → Assessment Activity**

Students complete BLM 9.1

Assign extending or enriching tasks before the students begin so that those students who finish early can quietly move on to these extensions. Those who do not have time to work on the extensions can do them outside of class.

Circulate as students work. If you notice a student who is not making progress, consider:

- giving the student manipulatives to build models of pools and patios;
- observing the student building or drawing the 4<sup>th</sup> diagram and ask prompting questions.

If a student is generating a low level response, ask prompting questions on the student's answer sheet, using colourful ink. The student responds in writing.

This written record of prompting questions and answers can be referred to during a student or parent interview.

**Curriculum Expectations/Written Work/Rubric:** Assess students demonstration of their learning, using a rubric.

For samples of student work refer to The Ontario Curriculum, exemplars, Grade 7, Mathematics.



See the sample rubric in the exemplar.

**Consolidate Debrief****Whole Class → Set Required Proficiency Level**

Collect completed student work. Distribute copies of a sample Developing Proficiency test based on the curriculum expectation: extend a pattern, complete a table, and write words to explain the pattern. See Section 2 – *Patterning to Algebraic Modelling* content-based package. Explain that all students need to develop proficiency on these sorts of questions and that each student will have the opportunity to try a similar test and get help if needed until they meet the required proficiency level.

After students have had a chance to complete a sample Developing Proficiency test independently, take up solutions and answer questions.

**Curriculum Expectations/Proficiency Test/Marking Scheme:** When it appears that most students in the class are ready to demonstrate their proficiency, give students another version of the test to do independently. Collect and score checking whether the proficiency target has been met or proficiency is still developing. When returning scored tests, provide another teaching loop for students who did not meet the target while those who met the target work on extensions.



When returning graded work to students, consider photocopying samples of Level 3 and 4 responses with student names removed. Select and discuss, with the class, samples that show a variety of strategies.

**Home Activity or Further Classroom Consolidation**

Pick a number between 5 and 10. Complete the multiplication facts for that number from 1 to 12. Identify patterns in the multiplication facts.

Exploration  
Skill Drill

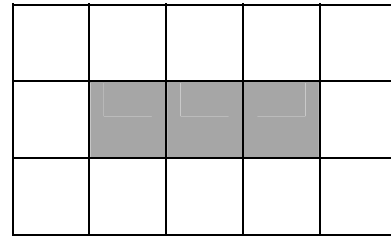
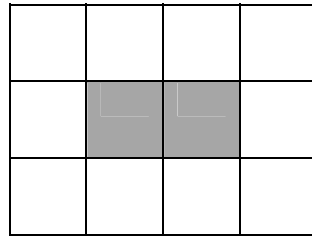
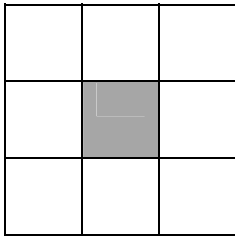
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## 9.1: Performance Task

**Name:**

**Date:**

1. A rectangular pool is surrounded by a patio. Below are the first three terms of a pattern for designing the pool. The dark squares represent the water in the rectangular pool and the light squares represent the patio.



- a) If the pool has an area of twenty-five square units, how many tiles are needed for the patio? Explain your thinking.
- b) If the pool has an area of  $n$  square units, how many tiles are needed for the patio? Show how you arrived at your answer.
- c) If there are 206 patio tiles, what size of a rectangular pool can you build? Explain your thinking.
2. a) Use two different coloured tiles and construct a different arrangement for a pool and a patio.
- b) Describe how you would find out how many tiles would be in the thirteenth pool.

Source: The Ontario Curriculum, Exemplars, Grade 7, Mathematics, 2002.